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INFLUENCE OF THE LINEAR DISTRIBUTION OF THE CHARGE-CARRIER DENSITY ALONG THE ARM OF A THERMOCOUPLE ON THE REGIME OF ITS OPERATION

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The problem on the temperature field in an inhomogeneous one-dimensional arm of a thermocouple has been solved by numerical methods with regard for both the distributed Peltier effect and the Thomson effect. It was assumed that the charge carriers are nondegenerate. The temperature field of the thermocouple was optimized for the regimes of maximum temperature drop and maximum refrigerating capacity. An optimum range of change in the charge-carrier density gradient has been determined.

The widespread use of thermoelectric temperature transducers in practice poses the problem of increasing their efficiency. This can be done first of all by increasing the thermoelectric Q of these devices. The values of this parameter attained to date in practice are far from the theoretically predicted limit [1]. As is known, a thermocouple with an arm with properties changing along its length has a higher thermoelectric Q than a thermocouple with a homogeneous arm [2]. An investigation of thermocouples with arms with properties changing along their length, as a limiting case of a complex thermocouple, [3] has shown that the thermoelectric Q of a thermocouple increases with increase in its conductivity and decrease in the thermal e.m.f. from the hot to the cold end of the arm. A distinctly different approach to the solution of this problem has been proposed in [4]. This approach is based on solution of the boundary problem on steady-state heat conduction. In this case, the optimum distribution of an impurity along the length of an arm is determined by solving the variational problem with the use of the Pontryagin maximum principle. However, in the case where this formalism is used, the problem should be substantially simplified. For example, in [4] it was assumed that the thermal e.m.f., the heat conduction, and the electrical conduction depend only slightly on the temperature and the Thomson effect can be disregarded, which significantly diminishes the usefulness of the results obtained. Moreover, only one regime was considered in this work — the regime of maximum temperature drop, while the regime of maximum refrigerating capacity is the most interesting from the practical standpoint. In the present work, we made an effort to solve the boundary problem of [4] with allowance for the temperature dependence of the kinetic coefficients and for the Thomson effect. Since the problem is nonlinear in this case, we numerically solved the boundary problem and numerically optimized the solution obtained. The regimes of maximum temperature drop and maximum refrigerating capacity were considered.

To determine the role of the Thomson effect, we solved both problems with and without regard for this effect. The last-mentioned case was considered earlier in [5]. The temperature field of the adiabatically isolated inhomogeneous one-dimensional arm of an unloaded thermocouple is defined, without regard for the Thomson effect, by the steady-state heat-conduction equation [4]

$$\frac{d}{dx}\left(\chi \frac{dT}{dx}\right) + \frac{y^2}{\sigma} - yT\frac{d\alpha}{dx} = 0 \tag{1}$$

with boundary conditions

$$\chi \frac{dT}{dx}\bigg|_{x=0} = \alpha yT\big|_{x=0}\,, \quad T\big|_{x=1} = T_1\,, \tag{2}$$

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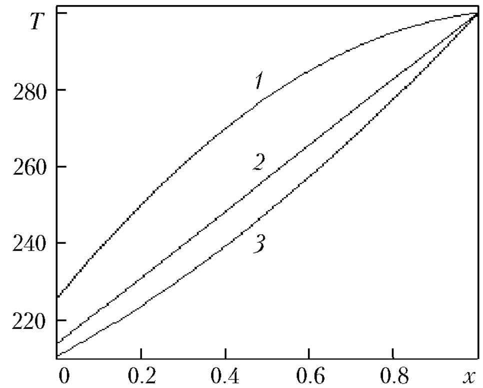

Fig. 1. Dependence of the temperature distribution along the arm of a thermocouple on the coefficient g characterizing the impurity distribution in the arm: $g = 0$ (1), 0.9 (2), and 0.999 (3).

into which, as differentiated from [4], we introduced, for convenience, the parameter $y = Jl/S$. In the nondegenerate case, the kinetic coefficients have the form

$$\sigma = enu\ ,\ \ \chi = \chi_{\text{lat}} + 2\left(\frac{k}{e}\right)^2 T\sigma\ ,\ \ \alpha = \frac{k}{e}\left(2 + \ln\frac{2\,(2\pi mkT)^{3/2}}{nh^3}\right).$$

The charge-carrier mobility u, the effective mass m, and the lattice heat conductivity χ_{lat} were selected such that the thermoelectric properties correspond to the properties of a semiconductor material with $Z = 3.0\cdot10^{-3}$ K^{-1} at $T_1 = 300$ K.

The parameter y introduced by us may be called the specific current. It is independent of the geometry of the arm and is determined by only the physical properties of its material and temperature. The optimum value of the specific current is determined by the optimum current if the length and the cross section of the arm are numerically equal. To determine the optimum current of an arm having a different geometry, it will suffice to multiply the specific current by the ratio S/l.

Equation (1) used for solving the boundary problem can be written, with regard for the Thomson effect, as

$$\frac{d}{dx}\left(\chi\frac{dT}{dx}\right) + \frac{y^2}{\sigma} + \frac{k}{e}yT\left(\frac{1}{n}\frac{dn}{dx} - \frac{3}{2T}\frac{dT}{dx}\right) = 0\ . \tag{3}$$

The carrier concentration is distributed along the arm, just as in [5], by the linear law

$$n = n_0\,(1 - gx)\ . \tag{4}$$

In solving the boundary problem, we numerically optimized the temperature drop with respect to the specific current and the density n_0 at the cold end of the thermocouple arm at a definite value of g. The range of change in this quantity $0 \le g \le 0.999$ corresponds to the range of change in the ratio between the carrier densities $c = n_0/n_1$ at the cold and hot ends of the arm $1 \le c \le 10^3$.

The results of numerical solution of boundary problems (1), (2) and (2), (3) are presented in the figures. Figure 1 presents the temperature distribution along thermocouple arms at optimum values of the specific current in the regime of maximum temperature drop. Curve 1 corresponds to a homogeneous arm ($g = 0$). In this case, the maximum temperature is reached at the hot end of the arm. Below are curves of temperature distribution in inhomogeneous arms with $g = 0.9$ and 0.999. If the charge carriers are distributed inhomogeneously, the maximum of the temperature dependence shifts outside the region considered due to the distributed Peltier effect. This dependence is linear in character at $g = 0.9$ and a curvature opposite in sign appears at $g = 0.999$. Due to the carrier density gradient, the temperature of the cold end of the arm additionally decreases as a result of the partial or complete compensation of the Joule heat. The optimization of the carrier density at the cold end of the arm leads to an increase in n_0. An in-

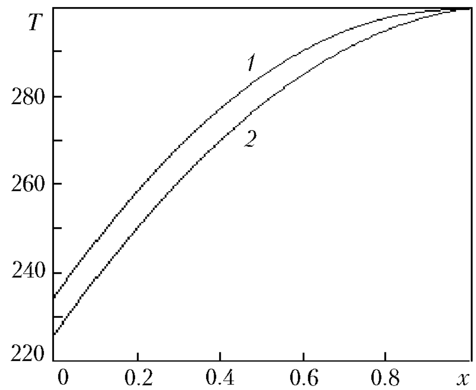

Fig. 2. Temperature distribution along the arm of a thermocouple determined without regard for the Thomson effect (1) and with regard for this effect (2).

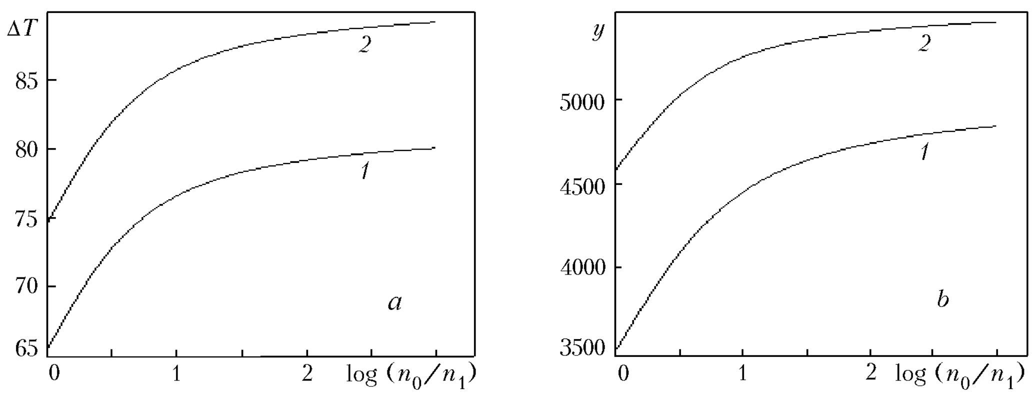

Fig. 3. Dependence of the maximum temperature drop (a) and the optimum value of the specific current (b) on the ratio between the charge-carrier densities at the cold and hot ends of the arm of a thermocouple determined without regard for the Thomson effect (1) and with regard for this effect (2).

crease in the optimum density of charge carriers at the cold end is an adverse factor, since it leads to a decreases in the thermal e.m.f. at this point and, consequently, to a partial decrease in the temperature drop. Since the Thomson effect was not taken into account in [5], it is interesting to consider its contribution to the processes studied. Figure 2 shows, for comparison, the temperature distributions obtained with and without regard for the Thomson effect.

The dependences of the maximum temperature drop along the length of the arm of a thermocouple on the logarithm of the ratio between the carrier densities at its cold and hot seals, obtained with and without regard for the Thomson heat, are presented in Fig. 3a. As is seen from these graphs, there is no point in changing the charge-carrier density by more than 8–10 times in the case of a linear temperature distribution because, at a large ratio between the carrier densities at the cold and hot ends of the arm, this will lead to a very small change in the temperature drop. For example, a tenfold change in the carrier density increases the temperature drop by 17% as compared to that of a homogeneous arm, and a thousandfold change in the carrier density increases the temperature drop by 22% in the case where the Thomson effect is not taken into account. When the Thomson effect is taken into account, the temperature drops increase by 15.5 and 20%, respectively. Figure 3b presents the dependences of the optimum value of the specific current on the logarithm of the ratio between the densities at the cold and hot seals of a thermocouple, obtained with and without regard for the Thomson effect. It is seen that larger temperature drops are attained at higher currents. A tenfold change in the carrier density increases the current by 26% and a thousandfold change in the carrier density increases the current by almost 40%.

The regime of maximum refrigerating capacity is of much greater importance in practice. It is known that, in this regime, the central region of the arm of a thermocouple is significantly overheated; therefore, it is very important

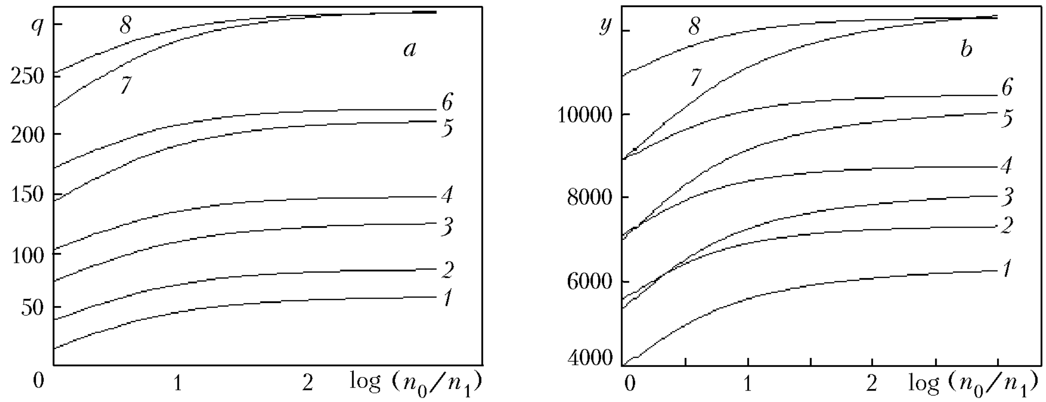

Fig. 4. Dependence of the specific maximum refrigerating capacity (a) and the optimum value of the specific current (b) on the ratio between the charge-carrier densities at the cold and hot ends of the arm of a thermocouple at a temperature drop of 60 K (1, 2), 40 K (3, 4), 20 K (5, 6), and 0 K (7, 8) determined without regard for the Thomson effect (1, 3, 5, 7) and with regard for this effect (2, 4, 6, 8).

to know the influence of the distributed Peltier effect and the Thomson effect on the temperature field of the thermocouple operating in this regime. In this case, the boundary condition at the cold end of the arm should have the form

$$\chi \left. \frac{dT}{dx} \right|_{x=0} = \alpha y T \big|_{x=0} - q\,, \tag{5}$$

where $q = Ql/S$ is the refrigerating capacity of the arm whose length and cross section are numerically equal to unity.

The solution of problem (3), (5) shows that the absorption of the heat released due to the distributed Peltier effect makes it possible to substantially decrease the overheating of the arm of a thermocouple or completely exclude it. Figure 4a shows the load characteristics of an arm at different temperature drops. It is seen that the specific refrigerating capacity of the arm increases by the largest value when the carrier density changes by 10–30 times. At a temperature drop of 60 K, a tenfold change in the carrier concentration causes the refrigerating capacity to increase by 3.3 times, and this capacity increases by 4.1 times when the carrier density changes by 25 times. As the temperature drop decreases, the multiplicity of increase in the refrigerating capacity decreases and, at a zero temperature drop, reaches 1.25 and 1.44 respectively. It should be noted that, when the temperature drop decreases (with increase in the load), the contribution of the Thomson effect becomes negligibly small for large density drops (curves 7 and 8). Figure 4b shows the dependence of the optimum value of the specific current on the logarithm of the ratio between the carrier densities at the cold and hot ends of the arm.

Thus, as the calculations show, the use of an inhomogeneous arm with a linear distribution of the carrier density along its length in a thermoelectric temperature transducer makes it possible to substantially increase the maximum temperature drop in it and, consequently, to increase the refrigerating capacity of the transducer. In this case, the Thomson effect should not be ignored, since its contribution is comparable to the contribution of the distributed Peltier effect.

NOTATION

e, elementary charge, C; *g*, proportionality coefficient; *h*, Planck constant, J·sec; *k*, Boltzmann constant, $J{\cdot}K^{-1}$; *l*, length of the arm of a thermocouple, m; *m*, effective mass of charge carriers, kg; *n*, density of charge carriers, m^{-3}; n_0, density of charge carriers at the cold end of the arm of a thermocouple, m^{-3}; n_1, density of charge carriers at the hot end of the arm of a thermocouple, m^{-3}; *Q*, refrigerating capacity of the arm of a thermocouple, W; *q*, specific refrigerating capacity, $W{\cdot}m^{-1}$; *S*, cross section of the arm of a thermocouple, m^2; *T*, temperature of the arm of a thermocouple as a function of a coordinate, K; T_1, temperature of the hot end of the arm of a thermocouple, K; *u*, mobility of charge carriers, $m^2 \cdot V^{-1} \cdot sec^{-1}$; x, dimensionless coordinate, $0 \le x \le 1$; y, specific current, $A \cdot m^{-1}$; Z, parameter of thermoelectric efficiency, K^{-1}; α, differential thermal e.m.f., $V \cdot K^{-1}$; σ, electrical conductivity, $\Omega^{-1} \cdot m^{-1}$; χ, heat conductivity, $W \cdot m \cdot K^{-1}$; χ_{let}, lattice heat conductivity, $W \cdot m \cdot K^{-1}$. Subscripts: lat, lattice.

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